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A STUDY OF STEADY LAMINAR BOUNDARY LAYER FLOW AND HEAT TRANSFER ALONG AN INFINITE, POROUS, HOT, VERTICAL CONTINUOUS MOVING PLATE IN THE PRESENCE OF HEAT SOURCE AND CONSTANT FREE STREAM .

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1.01 Abstract

An analysis is made for the study of two- dimensional laminar boundary layer flow of a viscous, incompressible fluid, along an infinite, porous, hot, vertical continuous moving plate. The governing partial differential equations are non-dimensionalized and are solved using Natural Transform Technique. The expressions for velocity field, temperature field, rate of heat transfer and skin-friction have been obtained. The influence of various physical parameters, such as Eckert number Ec, Prandtl number Pr, Grashoff number Gr, plate velocity α and heat source/sink parameters is extensively discussed with the help of graphs to show the physical aspects of the problem. It is found that these parameters significantly affect the flow and heat transfer.

Key Words: Incompressible fluid, Laminar flow, boundary layer, moving porous plate, heat transfer, natural transformation.

1.02 Introduction

A study of boundary layer (Sparrow [19], Schlichting [17], Bansal [3]) behavior on continuous solid surface has attracted the attention of researchers because such flows may find applications in different areas such as aerodynamic extrusion of plastic sheets, the boundary layer along material handling conveyers, the cooling of an infinite metallic plate in a cool bath etc. The classical problem was introduced by Blasius[5] by studying the boundary layer flow on a fixed flat plate. The flow field due to moving flat surface was developed by Sakiadis [16] where he took the constant velocity of plate. Crane [9] extended the work of Sakiadis[16] for two dimensional problem under some specific conditions. Yao, Fang and Zhong[21] studied the heat transfer of a generalized stretching/shrinking wall problem with convective boundary conditions. Cortell [8] investigated the flow and heat transfer of a viscoelastic fluid over a stretching sheet.

Prasad et.al.[15] studied the momentum and heat transfer in viscoelastic fluid flow in porous medium over a non –isothermal stretching sheet. Later Cortell [7] found the similarity solutions for flow and heat transfer of viscoelastic fluid over a porous stretching sheet. Vajravelu [20] obtained the solution of boundary layer flow and heat transfer over a continuous porous surface moving in an oscillating free stream.

The generalization of Laplace-Transform i.e. Natural-Transform, initially was defined by Khan and Khan [10] as N - transform, who studied their properties and applications. Later, Belgacem et al. [4] and Silambarasan et al. [18] defined its inverse and studied some additional fundamental properties of this integral transform and named it the Natural Transformation. Applications of Natural transform in the solution of partial differential equations were studied by Al-Omari [2] and Bulu et al. [6]. Loonker et al. [11,12] applied the Natural Transform for the distribution and Boehmians spaces. The recent development for the use of Natural Transform in the study of boundary layer on moving horizontal plate has been done by Agarwal et al. [1].

Aim of the present chapter is to investigate steady laminar boundary layer flow and heat transfer through an incompressible viscous fluid along an infinite, porous, hot vertical continuous moving plate in the presence of volumetric rate of heat generation (or absorption) and constant free stream by means of Natural Transformation.

1.03 Formulation of the Problem

Consider the steady boundary layer flow and heat transfer of a viscous incompressible fluid along an infinite hot vertical continuous moving plate in the presence of constant suction at the surface, constant free stream U_∞ and heat generation (or absorption). The plate is moving in upwards direction i.e.in flow direction with constant velocity and maintained at a constant temperature T_w. The flow is in positive direction of X*-axis in upwards direction and Y*-axis is taken normal to the plate.

The governing boundary layer equations of continuity, motion and energy for flow of an incompressible viscous fluid along an infinite, hot, vertical, porous continuous moving plate can be calculated as

Equation of Continuity

In two-dimension flow, the equation of continuity

$$\frac{\partial v^*}{\partial y^*} = 0 \Rightarrow v^* = -v_0 \text{(constant)},\ v_0 > 0 \qquad \text{... (1.03.01)}$$

Equation of Motion

The equation of motion in vertical direction i.e.in x- direction

$$\rho\left(-v_0 \frac{\partial u^*}{\partial y^*}\right) = \mu \frac{\partial^2 u^*}{\partial y^{*2}} + \rho g \beta \left(T^* - T_\infty\right) \qquad \text{... (1.03.02)}$$

Equation of Energy

The equation of energy in vertical direction

$$\rho C_p \left(-v_0 \frac{\partial T^*}{\partial y^*}\right) = \kappa \frac{\partial^2 T^*}{\partial y^{*2}} + \mu \left(\frac{\partial u^*}{\partial y^*}\right)^2 + Q\left(T^* - T_\infty\right) \qquad \text{... (1.03.03)}$$

where u*, v* are the velocity components along X*- axis and Y*- axis, respectively, ρ the density, v_0 the cross-flow velocity, μ the coefficient of viscosity, C_p the specific heat at constant pressure, κ the thermal conductivity, Q the volumetric rate of heat generation parameter, g the acceleration due to gravity, T_∞ the free stream temperature and β the volume expansion.

The corresponding boundary conditions are

$$\left.\begin{array}{l} y^* = 0 \quad : u^* = U_w,\ v^* = -v_0,\ T^* = T_w \\ y^* \to \infty : u^* \to U_\infty,\ T^* \to T_\infty \end{array}\right\} \qquad \text{... (1.03.04)}$$

1.04 Method of Solution

Introducing the following non-dimensional quantities

$$\left.\begin{aligned} & y = y^* \frac{v_0}{\upsilon}, u = \frac{u^*}{U_\infty}, \theta = \frac{T^* - T_\infty}{T_w - T_\infty}, \alpha = \frac{U_w}{U_\infty} \\ & \Pr = \frac{\mu C_p}{\kappa}, S = \frac{Qv^2}{\kappa v_0^2}, Ec = \frac{U_\infty^2}{C_p\left(T_w - T_\infty\right)}, Gr = \frac{g\beta\upsilon}{U_\infty v_0^2}\left(T_w - T_\infty\right) \end{aligned}\right\} \quad \text{... (1.04.01)}$$

into the equation (1.03.02) and (1.03.03), to get

$$u'' + u' = -Gr\theta \quad \text{… (1.04.02)}$$

$$\theta'' + \Pr\theta' + S\theta = -Ec\Pr(u')^2 \quad \text{… (1.04.03)}$$

Where Pr is the Prandtl number, Ec the Eckert number, S the heat source/sink parameter and dashes denote the differentiation w.r.t. y.

The boundary conditions in non- dimensional form are

$$\left.\begin{aligned} & y = 0: u = \alpha, \theta = 1 \\ & y \to \infty: u \to 1, \theta \to 0 \end{aligned}\right\} \quad \text{... (1.04.04)}$$

Where ’α’ is the velocity ratio parameter.

The equations (1.04.02) and (1.04.03) are ordinary non-linear second order coupled differential equations with constant coefficients and solved under the boundary conditions (1.04.04).

For incompressible fluid flow, the Eckert number is very small therefore u(y) and θ(y) can be expanded in the powers of Ec as given below-

$$u(y) = u_0 + Ecu_1 + O(Ec^2) \quad \text{... (1.04.05)}$$

$$\theta(y) = \theta_0 + Ec\theta_1 + O(Ec^2) \quad \text{... (1.04.06)}$$

Comparing the coefficients of like powers of Ec, we get

Zeroth-order Equations

$$u''_0 + u'_0 = -Gr\theta_0 \quad \text{… (1.04.07)}$$

$$\theta''_0 + \Pr\theta'_0 + S\theta_0 = 0 \quad \text{… (1.04.08)}$$

First-order Equations

$$u''_1 + u'_1 = -Gr\theta_1 \quad \dots (1.04.09)$$

$$\theta''_1 + \Pr\theta'_1 + S\theta_1 = -\Pr\left(u'_0\right)^2 \quad \dots (1.04.10)$$

The corresponding boundary conditions are

$$\left.\begin{array}{l} y=0 \quad : \; u_0=\alpha, u_1=0, \theta_0=1, \theta_1=0 \\ y\to\infty : \; u_0\to 1, u_1\to 0, \theta_0\to 0, \theta_1\to 0 \end{array}\right\} \quad \dots (1.04.11)$$

Now we use the Natural Transform to solve the zeroth order and first order equations. Solving equation (1.04.08), we get

$$\theta_0(y) = \frac{1}{2}\left\{ e^{\frac{-\Pr+\sqrt{\Pr^2-4S}}{2}y} + e^{\frac{-\Pr-\sqrt{\Pr^2-4S}}{2}y} \right\}$$

$$+\frac{1}{\sqrt{\Pr^2-4S}}\left\{\theta'_0(0)+\frac{1}{2}\Pr\right\}\left\{ e^{\frac{-\Pr+\sqrt{\Pr^2-4S}}{2}y} - e^{\frac{-\Pr-\sqrt{\Pr^2-4S}}{2}y} \right\}$$

Using boundary condition at y→∞, to give

$$\theta'_0(0) = -\frac{\Pr+\sqrt{\Pr^2-4S}}{2}$$

This provides

$$\theta_0(y) = e^{\frac{-\Pr-\sqrt{\Pr^2-4S}}{2}y} = e^{m_2 y}, \text{ where } m_2 = \frac{-\Pr-\sqrt{\Pr^2-4S}}{2} \quad \dots (1.04.12)$$

Therefore equation (1.04.07) becomes

$$u''_0 + u'_0 = -Gre^{m_2 y}$$

Using Natural Transformation and after little simplification, we get

$$u_0(y) = \frac{Gr}{m_2} - \frac{Gr}{(1+m_2)}e^{-y} - \frac{Gr}{m_2(1+m_2)}e^{m_2 y} + \left(\alpha + u'_0(0)\right) - u'_0(0)e^{-y}$$

Now applying boundary condition at y→∞, to give

$$u'_0(0)=1-\alpha-\frac{Gr}{m_2}$$

Hence,

$$u_0(y)=1+A_1e^{-y}-B_1e^{m_2y}, \quad \text{...(1.04.13)}$$

Where $A_1=\alpha-1+\dfrac{Gr}{m_2(1+m_2)}$ and $B_1=\dfrac{Gr}{m_2(1+m_2)}$ ... (1.04.14)

Now we solve first order equations.

Using equation (1.04.13) into equation (1.04.10), we get

$$\theta''_1+\Pr\theta'_1+S\theta_1+\Pr\left(-A_1e^{-y}-B_1m_2e^{m_2y}\right)^2=0$$

Solving it, using Natural Transform, we get

$$\theta_1(y)=\frac{2\theta'_1(0)e^{-\frac{\Pr}{2}y}}{\sqrt{\Pr^2-4S}}\left\{\frac{e^{\frac{y}{2}\sqrt{\Pr^2-4S}}-e^{-\frac{y}{2}\sqrt{\Pr^2-4S}}}{2}\right\}$$
$$-\left[\frac{A_1^2\Pr}{(S-2\Pr+4)}e^{-2y}+\frac{2A_1^2\Pr}{\left(\Pr+\sqrt{\Pr^2-4S}-4\right)\sqrt{\Pr^2-4S}}e^{m_2y}+\frac{2A_1^2\Pr}{\left(-\Pr+\sqrt{\Pr^2-4S}+4\right)\sqrt{\Pr^2-4S}}e^{-(\Pr+m_2)y}\right]$$
$$-\left[-\frac{2B_1^2\Pr m_2}{\left(\Pr+3\sqrt{\Pr^2-4S}\right)}e^{2m_2y}+\frac{B_1^2\Pr m_2}{\sqrt{\Pr^2-4S}}e^{m_2y}+\frac{2B_1^2\Pr m_2^2}{\left(\Pr+3\sqrt{\Pr^2-4S}\right)\sqrt{\Pr^2-4S}}e^{-(\Pr+m_2)y}\right]$$
$$-\left[\frac{2A_1B_1\Pr m_2}{\left(1+\sqrt{\Pr^2-4S}\right)}e^{(m_2-1)y}-\frac{2A_1B_1\Pr m_2}{\sqrt{\Pr^2-4S}}e^{m_2y}+\frac{2A_1B_1\Pr m_2}{\left(1+\sqrt{\Pr^2-4S}\right)\sqrt{\Pr^2-4S}}e^{-(\Pr+m_2)y}\right]$$

After applying boundary condition at y→∞, we get

$$\theta'_1(0)=\frac{2A_1^2\,\mathrm{Pr}}{(-\mathrm{Pr}+4+\sqrt{\mathrm{Pr}^2-4S})}+\frac{2B_1^2\,\mathrm{Pr}\,m_2^2}{(\mathrm{Pr}+3\sqrt{\mathrm{Pr}^2-4S})}+\frac{2A_1B_1\,\mathrm{Pr}\,m_2}{(1+\sqrt{\mathrm{Pr}^2-4S})}$$

This further gives

$$\theta_1(y)=A_2e^{-2y}+B_2e^{m_2y}+C_2e^{(m_2-1)y}+G_2e^{2m_2y} \qquad \text{... (1.04.15)}$$

Where

$$A_2=-\frac{A_1^2\,\mathrm{Pr}}{\left(S-2\,\mathrm{Pr}+4\right)},\; C_2=\frac{-2A_1B_1\,\mathrm{Pr}\,m_2}{1-\left(\mathrm{Pr}+2m_2\right)},\;\; G_2=\frac{-B_1^2\,\mathrm{Pr}\,m_2}{\mathrm{Pr}+3m_2}\;\text{ and}$$

$$B_2=-\left(A_2+G_2+C_2\right) \qquad \text{... (1.04.16)}$$

Next we take equation (1.04.09),

$$u''_1+u'_1=-Gr\theta_1$$

$$\Rightarrow u''_1+u'_1=-Gr\left[A_2e^{-2y}+B_2e^{m_2y}+C_2e^{(m_2-1)y}+G_2e^{2m_2y}\right]$$

Solving it by using Natural Transform, we get

$$u_1(y)=u'_1(0)\left[1-e^{-y}\right]-\frac{Gr}{2}\begin{bmatrix} A_2\left(1-2e^{-y}+e^{-2y}\right) \\ +2B_2\left(-\frac{1}{m_2}+\frac{1}{(m_2+1)}e^{-y}+\frac{1}{m_2(m_2+1)}e^{m_2y}\right) \\ +2C_2\left(-\frac{1}{(m_2-1)}+\frac{1}{m_2}e^{-y}+\frac{1}{m_2(m_2-1)}e^{(m_2-1)y}\right) \\ +G_2\left(-\frac{1}{m_2}+\frac{2}{(2m_2+1)}e^{-y}+\frac{1}{m_2(2m_2+1)}e^{2m_2y}\right)\end{bmatrix}$$

After applying boundary condition at y → ∞, we get

$$u'_1(0)=\frac{Gr}{2}\left[A_2-\frac{2B_2}{m_2}-\frac{2C_2}{(m_2-1)}-\frac{G_2}{m_2}\right]$$

which gives

$$u_1(y)=A_3e^{-2y}+B_3e^{m_2y}+C_3e^{(m_2-1)y}+G_3e^{2m_2y}+H_3e^{-y} \qquad \text{... (1.04.17)}$$

Where

$$A_3 = -\left(\frac{GrA_2}{2}\right),\quad B_3 = -\left(\frac{GrB_2}{m_2\left(m_2+1\right)}\right),\quad C_3 = -\left(\frac{GrC_2}{m_2(m_2-1)}\right),\quad G_3 = -\left(\frac{GrG_2}{2m_2\left(2m_2+1\right)}\right)$$

and $H_3 = -(A_3 + B_3 + C_3 + G_3)$ ... (1.04.18)

Hence the velocity field (1.04.05) and temperature field (1.04.06) become

$$u(y) = u_0 + Ecu_1 + O(Ec^2)$$

$$u(y) = 1 + A_1 e^{-y} - B_1 e^{m_2 y} + Ec\left\{A_3 e^{-2y} + B_3 e^{m_2 y} + C_3 e^{(m_2-1)y} + G_3 e^{2m_2 y} + H_3 e^{-y}\right\} \quad \text{... (1.04.19)}$$

$$\theta(y) = \theta_0 + Ec\theta_1 + O(Ec^2)$$

$$\theta(y) = e^{m_2 y} + Ec\left\{A_2 e^{-2y} + B_2 e^{m_2 y} + C_2 e^{(m_2-1)y} + G_2 e^{2m_2 y}\right\} \quad \text{... (1.04.20)}$$

Neglecting higher order terms of Ec.

1.05 Skin-Friction Coefficient

The skin-friction coefficient C_f at the plate is given by

$$C_f = \frac{\tau_w}{\rho U_\infty v_0} = \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$C_f = -A_1 - m_2 B_1 + Ec\left\{-2A_3 + m_2 B_3 + (m_2-1)C_3 + 2m_2 G_3 - H_3\right\} \quad \text{... (1.05.01)}$$

1.06 Nusselt Number

The rate of heat transfer in terms of Nusselt number at the plate is given by

$$N_u = \frac{v}{v_0}\frac{q}{\kappa(T_w - T_\infty)} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$

$$N_u = -m_2 - Ec\left\{-2A_2 + m_2 B_2 + (m_2-1)C_2 + 2m_2 G_2\right\} \quad \text{... (1.06.01)}$$

1.07 Result and Discussion

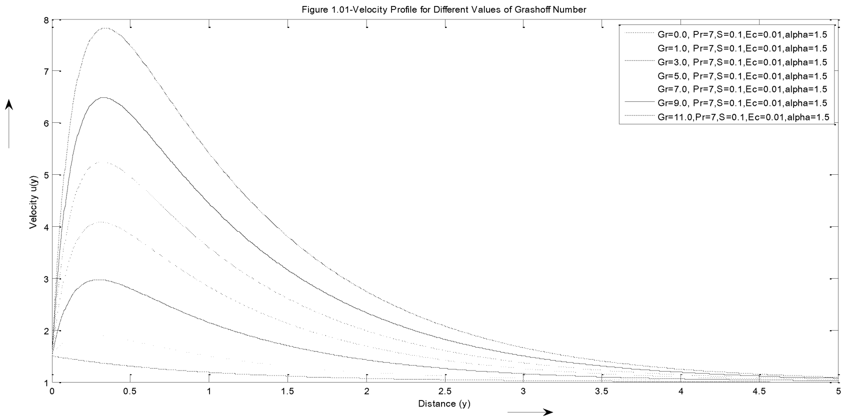

Fig.1.01

From this figure it is clear that for zero Grashoff Number (i.e. when the fluid temperature at wall is same as the free stream temperature or $T_w = T_\infty$) the fluid velocity decreases continuously with distance (y), but for positive Grashoff Numbers the velocity attains its maximum value at y=0.5 (approx.) and then it decreases exponentially. We can also observe that mean fluid velocity increases with increasing value of the Grashoff Number.

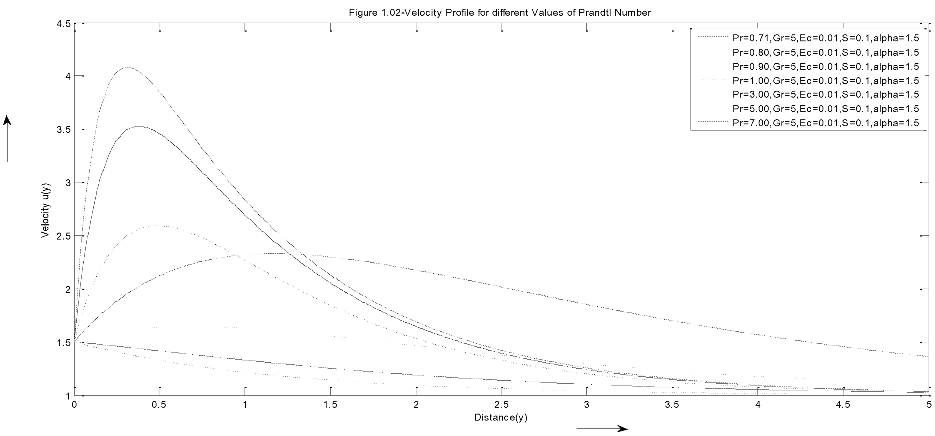

Fig.1.02

From this figure we observe that the mean stream velocity is minimum for Prandtl Number one (Pr=1).For Pr >1 higher the Prandtl Number greater will the velocity. The phenomena get reversed for Pr <1, i.e. as the Prandtl Number of fluid decreases the mean velocity increases.

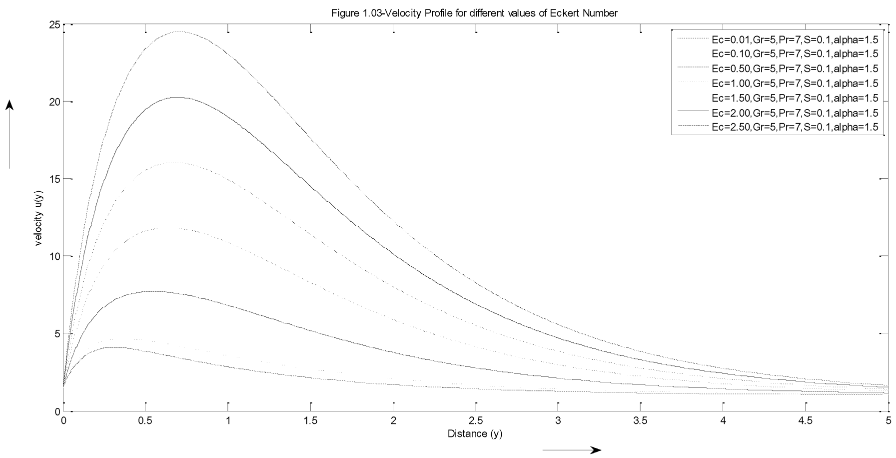

Fig.1.03

It is evident from this figure that the mean fluid velocity increases with increasing the Eckert Number. i.e. under the same circumstances the fluid with large Eckert Number flows fast as compare to the fluid with smaller Eckert Number.

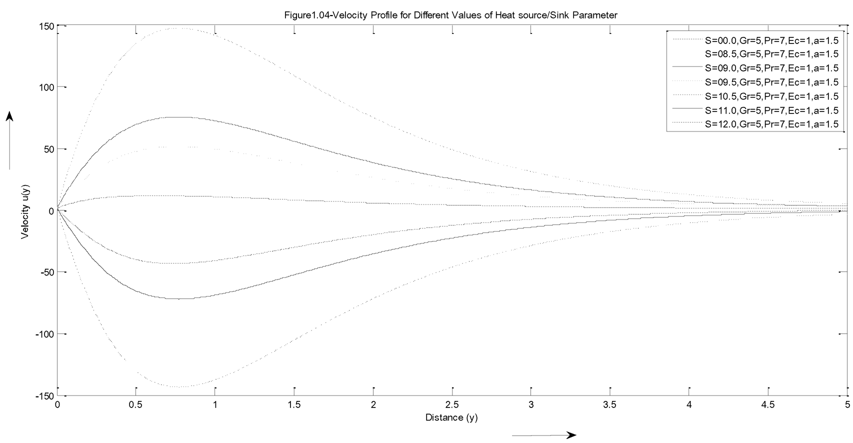

Fig.1.04

For the given set of values, we observe that the mean velocity increases with increasing the value of heat source parameter for S <10, and the direction of flow becomes opposite for S >10. The velocity decreases gradually as we increases the value of S (for S >10).

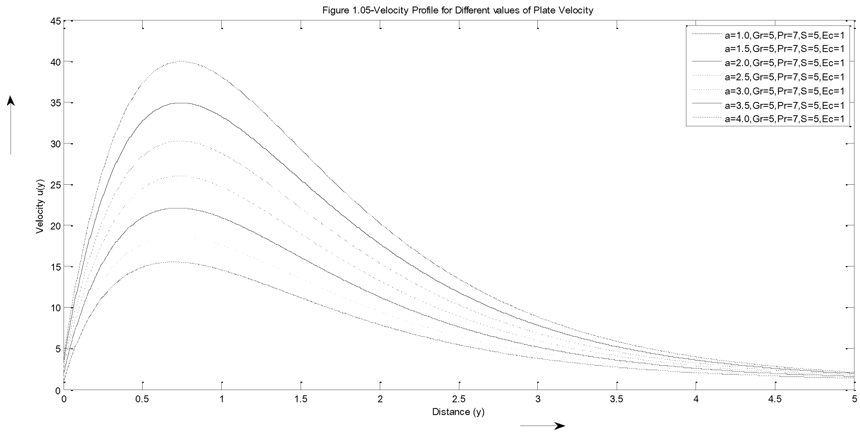

Fig.1.05

It is evident from this figure that the mean fluid velocity increases with increasing the plate velocity. Also for y=0.7(approx.) the fluid gets its maximum velocity. The graph between distance and velocity is skewed in right and changes its nature from platykurtic to leptokurtic as we increase the plate velocity.

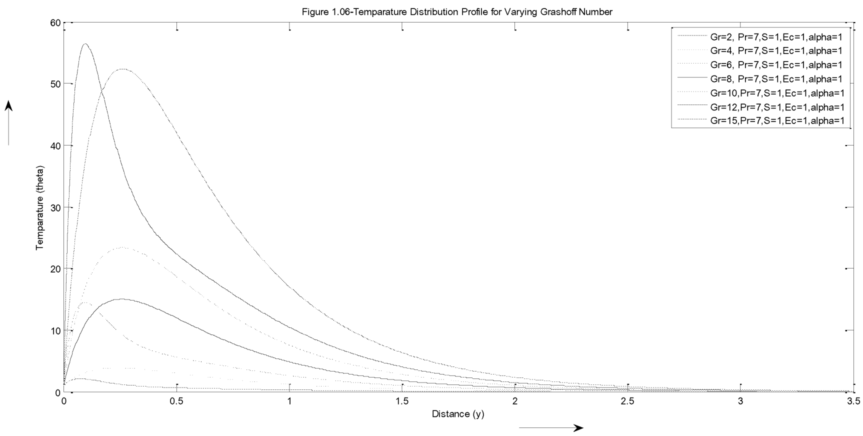

Fig.1.06

The above figure shows that as we increase the Grashoff Number of the fluid, the mean temperature distribution increases in the thin boundary layer near the plate where the viscous forces are confined.

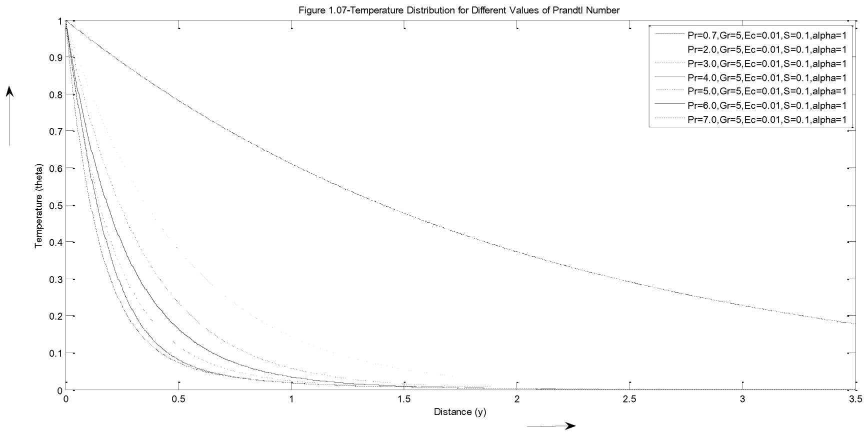

Fig.1.07

From this figure we observe that the temperature distribution decreases as we increase the value of Prandtl number. For the given set of data it is maximum for Pr=0.7 i.e. for air.

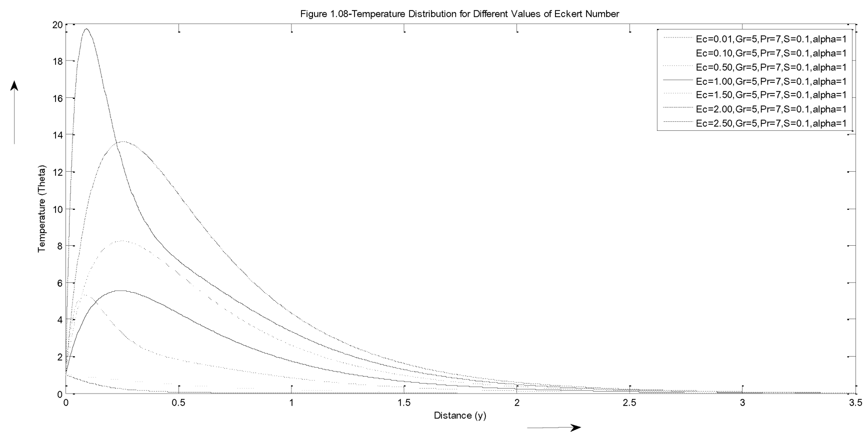

Fig.1.08

It is evident from this figure that the temperature distribution increases with increasing the value of Eckert Number. For very small Eckert Number the temperature decreases continuously with distance from plate.

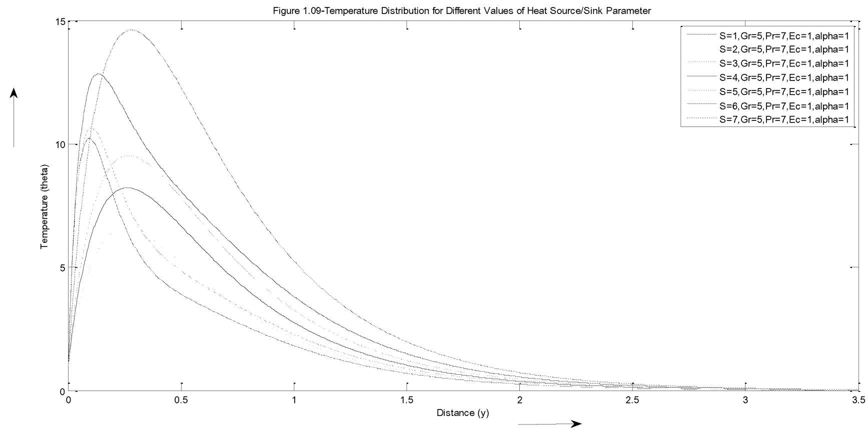

Fig.1.09

The above figure shows that as we increase the value of heat source parameter, the mean temperature distribution increases and sharp changes take place for large value of heat source parameter.

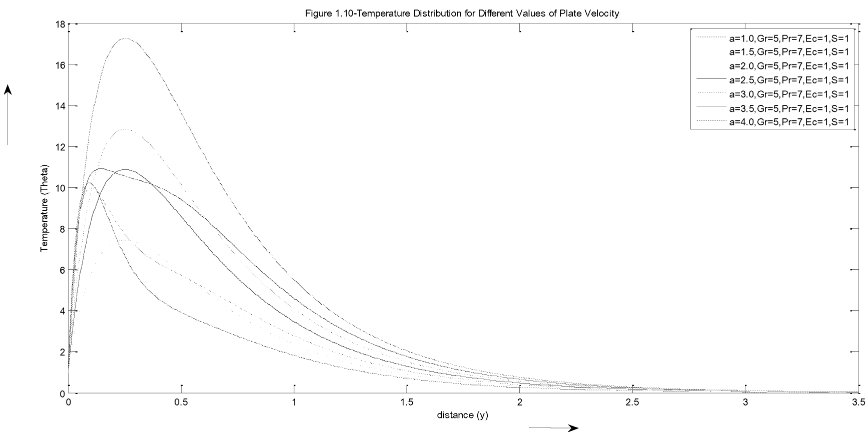

Fig.1.10

It is evident from this figure that the mean temperature distribution increases with increasing the plate velocity. From this phenomenon we conclude that more the plate velocity greater will the heat transfer.

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